

DEPARTMENT OF MATHEMATICS
HONOURS I METHODS OF CALCULUS
LIMITS AND CONTINUITY

1. Evaluate the following limits:

$$(a) \lim_{x \rightarrow \infty} \frac{(3x+1)(2x-1)(x-1)}{x^3}, \quad (b) \lim_{x \rightarrow \infty} x[\sqrt{(x^2+3)}-x],$$

$$(c) \lim_{x \rightarrow \frac{1}{2}\pi} \frac{1 + \cos x}{1 - \sin x}, \quad (d) \lim_{x \rightarrow 1} \frac{x^3+3x^2-2x-2}{2x^2-x-1},$$

$$(e) \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}, \quad (f) \lim_{x \rightarrow 3^+} \frac{x}{3-x}.$$

$$(g) \lim_{x \rightarrow 3^-} \frac{x}{3-x}$$

2. Determine the domains for which these functions are continuous:

$$(a) f(x) = x+|x|, \quad (b) f(x) = 1/(x^2-1),$$

$$(c) f(x) = \frac{x^5+x^2-1}{4+\sin x-2 \cos x}, \quad (d) f(x) = \frac{x^3+4x^2+x-6}{(x-1)(x+4)}.$$

3. Use L'Hospital's rule to evaluate the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{\tan 2x}{x}, \quad (b) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x - \sin x},$$

$$(c) \lim_{x \rightarrow 1} \frac{x^3-2x^2-x+2}{x^3-7x+6}, \quad (d) \lim_{x \rightarrow \frac{1}{2}\pi} \frac{\tan x}{\tan 5x},$$

$$(e) \lim_{x \rightarrow 0} \frac{\cot x}{x - \cot x}, \quad (f) \lim_{x \rightarrow \frac{1}{2}\pi} (\sin x)^{\tan x}.$$

Continued.

4. Show how to find

$$\lim_{x \rightarrow 0^+} x \ln x$$

given that

$$\lim_{x \rightarrow \infty} \frac{e^x}{x} = \infty.$$

Using these results and the fact that

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

evaluate the following limits

$$(i) \lim_{y \rightarrow 0} \frac{\tan^2 ay}{y^2}, \quad (ii) \lim_{x \rightarrow 0^+} x^2 \ln\left(\frac{1}{x}\right),$$

$$(iii) \lim_{x \rightarrow \infty} x^2 \ln\left(\frac{1}{x}\right), \quad (iv) \lim_{x \rightarrow 0^+} x^x.$$