

DEPARTMENT OF MATHEMATICS  
HONOURS I METHODS OF CALCULUS  
FUNCTIONS AND THEIR INVERSES

1. Specify the domains and ranges you would expect to be associated with the following functions.

(i)  $x\sqrt{x^2-1}$ , (ii)  $\cot x$ , (iii)  $\ln(\sin x)$ ,

(iv)  $\frac{1}{x^2}$ , (v)  $\frac{1}{x(x-1)}$ , (vi)  $\frac{x^2-1}{x-1}$ .

Sketch the graphs of the functions.

2. State which of the following functions are equal to the function

$$f(x) = |x - 2|.$$

Give reasons for your answers.

(a)  $s(x) = (x^2 - 4x + 4)^{\frac{1}{2}}$ ,

(b)  $t(x) = (x - 2)^{\frac{3}{2}} / (x - 2)^{\frac{1}{2}}$ ,

(c)  $u(x) = \frac{|x - 2|^2}{|x - 2|}$ ,

(d)  $v(x) = \frac{|6x - 12|}{6}$ ,

(e)  $w(x) = \left| \frac{x^2 + x - 6}{x + 3} \right|$ .

In each case the domain is the largest set of real numbers for which the formula make sense.

3. Sketch the graphs of the following functions for

$$-3 \leq x \leq 3.$$

(a)  $h(x) = \frac{[x]}{x}$ ,  $x \neq 0$ ;      (b)  $g(x) = [x] + [-x]$ ,

where  $[x]$  is the greatest integer less or equal to  $x$ .

4. What is the range of a function  $f: x \rightarrow \sin x$

(i) when the domain is the set of numbers  $-\frac{1}{2}\pi \leq x \leq \frac{1}{4}\pi$ ,

(ii) when the domain is the set of numbers  $0 \leq x \leq \pi$ ?

Sketch the graphs of these two functions.

5. Give the (simplified) numerical values of

$$\sin^{-1} \frac{1}{2}, \cos^{-1}(\cos \pi/3), \cos(\operatorname{cosec}^{-1} 2), \sec^{-1}(\tan \pi/4),$$

$$\exp(\ln 2), \ln(\exp 7), \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{3}{4}.$$

6. Why would  $[0, \pi]$  be a suitable range for  $\cos^{-1} x$ ? State the domain.

Repeat for  $[-\pi/2, \pi/2]$  with  $\tan^{-1} x$ .

Are these the only possible ranges? - if not, suggest others.

7. Find the inverse functions of the following functions.

(i)  $f: x \rightarrow (x-1)^3$ , all  $x$ .

(ii)  $g: x \rightarrow (x-1)^2 + 2$ ,  $x < 1$ .

(iii)  $y = \log(1+x) - \log(1-x)$ ,  $|x| < 1$ .

(iv)  $y = \tan^{-1} \left[ \frac{1 + \sqrt{3}x}{\sqrt{3-x}} \right]$ ;  $x \neq \sqrt{3}$ .

(v)  $h(x) = (\cos x - \sin x)^{-1}$ ,  $0 \leq x < \pi/4$ .